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SET THEORY FOR FRESHMAN STUDENT

FROM EXTREME GRADE 9 - 10

wollo university

kombolcha campus

2016 E.C

1.4 Greatest Common Factor (GCF)

The **greatest common factor (GCF)** of two or more number is the greatest number that divides each of them completely.

There are **two** methods for finding the **GCF** of two or more numbers.

i) **Factorization method**
ii) **Division algorithm method**

1.4.1 Using Prime factorization method (Fundamental theorem of Arithmetic)

To find **GCF** of two or more natural numbers:

✓ First find the prime factorization of the given numbers.
✓ Identify each common prime factor of the given numbers.
✓ Thus, **GCF** is the product of common prime factors with lowest powers.

Illustrative Example

8. Find each of the following
   a) GCF (72, 108)
   b) GCF (0, 15)
   c) GCF (84, 90)
   d) GCF (36, 135)

Solution: Refer 1.4.1

a) $72 = 2\times2\times2\times3\times3 = 2^3\times3^2$ } Lowest power of 2 is 2^2 and
$108 = 2\times2\times3\times3\times3 = 2^2\times3^3$ } lowest power of 3 is 3^2.

GCF is product of common prime factors with lowest powers $= 2^2\times3^2$.

$\therefore$ **GCF (72, 108) $= 2^2\times3^3 = 36$**

b) $0 = 0\times15$, $15 = 1\times15$ } $\therefore$ **GCF(0,15) = 15**

c) $84 = 2\times2\times3\times7 = 2^2\times3^1\times7$ } Lowest power of 2 is 2^1 and
$90 = 2\times3\times3\times5 = 2^1\times3^2\times5$ } lowest power of 3 is 3^1

$\therefore$**GCF (84, 90) $= 2^1\times3^1 = 6$**

d) $36 = 2\times2\times3\times3 = 2^2\times3^2$
$135 = 3\times3\times3\times5 = 3^3\times5$
$\therefore$ **GCF (36, 135) $= 3^2 = 9$**

9. If the prime factorization of $x = 2^5.3^4.5^2.7$ and $y = 2^3.5^3.11$, and $z = 2^2.3^3.7^2$ then
   a) GCF (x, y)
   b) GCF (x, z)
   c) GCF (x, y, z)

Solution:

a) GCF $(x, y) = 2^3.5^2 = 200$

b) GCF $(x, z) = 2^2.3^3.7 = 756$

c) GCF $(x, y, z) = 2^2 = 4$

10. Find two numbers that lies between 150 and 215 such that their GCF = 34.

Solution: $150 < 34d < 215$

Let $d = 4$, then $34d = 34(4) = 136$

Let $d = 5$, then $34d = 34(5) = 170$

Let $d = 6$, then $34d = 34(6) = 214$

Let $d = 7$, then $34d = 34(7) = 248$

∴ The two numbers are 170 and 214.

1.4.2 Using Euclid's Division Algorithm

For every pair of two positive integers **a** and **b**, there exist two whole numbers **q** and **r** such that $a = bq + r$, where $0 \leq r < b$

✓ **GCF (a, b) = GCF (b, r)**

In the relation $a = bq + r$, with $0 \leq r < b$ is long division of number, **a**, by **b**, in which, **q**, is the quotient obtained and, **r**, is the remainder.

Thus, **dividend = divisor ×quotient + remainder** $\Leftrightarrow a = bq+r$, with $0 \leq r < b$.

Important Note:

To find the **GCF**, of two positive integers **a** and **b**, where $a > b$

i) Divide **a** by **b** to get **q** and **r** so that $a = bq + r$, with $0 \leq r < b$.

ii) If $r = 0$, then GCF $(a, b) = (b, r) = b$

iii) If $r \neq 0$, divide **b** by **r** and continue in the same way till the remainder is zero.

Illustrative Example

11. Using Euclid's division algorithm find the value of q and r such that:

a. $215 = 11q + r$ b. $418 = 33q + r$

Solution:

a. As: $a = 215$ and $b = 11$

Divide 215 by 11.

$$\begin{array}{r} 19 \leftarrow q \\ 11\overline{)\,215} \\ -209 \\ \hline = 6 \leftarrow r \end{array}$$

∴ $215 = 11(19) + 6$

∴ $q = 19$ **and** $r = 6$

b. $a = 418$, and $b = 33$

Divide 418 by 33.

$$\begin{array}{r} 12 \leftarrow q \\ 33\overline{)\,418} \\ -396 \\ \hline = 22 \leftarrow r \end{array}$$

∴ $418 = 33(12) + 22$

∴ **q = 12, and r = 22**

12. Using Euclid's division algorithm, find the **G. C. F** of:
a. 224 and 84 b. 963 and 657
c. 3997 and 2947 d. 867 and 255

Solution: Refer # 1.4.2
Divide the larger number by smaller.

a. $224 = 84 \times 2 + 56$ ⟵ *Divide 224 by 84, keep r = 56*
$84 = 56 \times 1 + 28$ ⟵ *Divide 84 by 56, keep r = 28*
$56 = 28 \times 2 + 0$ ⟵ *Apply GCF (a, b) = GCF (b, r) = b if r = 0*
∴ **GCF (224, 84) = GCF (56, 28) = 28.**

b. $963 = 657 \times 1 + 306$ ⟵ *Divide 963 by 657, keep r = 306*
$657 = 306 \times 2 + 45$ ⟵ *Divide 657 by 306, keep r = 45*
$306 = 45 \times 6 + 36$ ⟵ *Divide 306 by 45, keep r = 36*
$45 = 36 \times 1 + 9$ ⟵ *Divide 45 by 36, keep r = 9*
$36 = 9 \times 4 + 0$ ⟵ *Apply GCF (a, b) = GCF (b, r) = b, if r = 0*
∴ **GCF (963, 657) = GCF (36,9) = 9**

c. $3997 = 2947 \times 1 + 1050$ ⟵ *Divide 3997 by 2947*
$2947 = 1050 \times 2 + 847$ ⟵ *Divide 2947 by 1050, keep r = 847*
$1050 = 847 \times 1 + 203$ ⟵ *Divide 1050 by 847, keep, r = 203*
$847 = 203 \times 4 + 35$ ⟵ *Divide 847 by 203, keep r = 35*
$203 = 35 \times 5 + 28$ ⟵ *Divide 203 by 35, keep r = 28*
$35 = 28 \times 1 + 7$ ⟵ *Divide 35 by 28, keep r = 7*
$28 = 7 \times 4 + 0$ ⟵ Apply GCF (a, b) = GCF (b, r) = b, if r = 0
∴ **GCF (3997, 2947) = GCF (28, 7) = 7.**

d. Exercise left for you

For what value of **r**, GCF (375, 225) = GCF (75, r)
A. 0 B. 3 C. 150 D. 1

Solution: Using Euclid's algorithm
We have, GCF (a, b) = GCF (b, r) = b if r = 0.

$375 = 225\ (1) + 150$
$225 = 150\ (1) + 75$
$150 = 75\ (2) + 0$

∴ GCF (375, 225) = GCF (225, 150)
= GCF (150, 75) = GCF (75,0) = 75

Answer: A

4. There are between 50 and 60 eggs in a basket. If one counts them by 3's, he has 2 eggs left over but if he counts them by 5, he has 4 egg left over. How many eggs are there in the basket … **CBE.**
A. 56 B. 52 C. 59 D. 57

Solution:
✓ If you
✓ If you

15. The maximum
pencils can be
same number o
A. 4
Solution: Requ
657 =
306 =
45 =
36 =

16. The GCF o
numbers?
Explanation: I

1.5.

The least commo
which is commo

Example: To fi
$M_6 = \{6,$
$M_9 = \{9,$
✓ The set of c
Because the
written **LC**
✓ The **prime**
find the LC
To find the L
• take th
either

Solution:

✓ If you divide 59 by 3, we get: $59 = 3(19) + 2$

✓ If you divide 59 by 5, we get $59 = 5(11) + 4$

Answer: C

15. The maximum number of students among them 306 pens and 657 pencils can be distributed in such a way that each student gets the same number of pens and same number of pencils is:

A. 4 B. 9 C. 36 D. 45

Solution: Required the GCF (657, 306)

$$\left.\begin{aligned} 657 &= 306(2) + 45 \\ 306 &= 45(6) + 36 \\ 45 &= 36(1) + 9 \\ 36 &= 9(4) + 0 \end{aligned}\right\} \quad GCF(657,306) = GCF(36,9) = 9$$

Answer: B

16. The GCF of two numbers is 1. What is the special name of the two numbers?

Explanation: If GCF (a, b) =1, then a and b are called **Relative prime**

1.5. LEAST COMMON MULTIPLE (LCM)

The least common multiple **(LCM)** of **a** and **b** is the smallest natural number which is common to multiple of **a** and multiple of **b**.

Example: To find the LCM of 6 and 9, denoted by M_6 and M_9 then:

$M_6 = \{6, 12, 18, 24, 30, 36, 42, \ldots\}$

$M_9 = \{9, 18, 27, 36, 45, 54, \ldots\}$

✓ The set of common multiples of 6 and 9 are $M_6 \cap M_9 = \{18, 36, 54, \ldots\}$ Because the least number in $M_6 \cap M_9$ is **18**, the LCM of 6 and 9 is 18, written **LCM (6, 9) = 18.**

✓ The **prime factorization Method** is another, more efficient method for find the LCM of several numbers

To find the LCM of a number:

- take the product of the **highest** power of each prime that occurs in either of the given numbers.

UNIT ONE: STUDY SKILLS

1.1.Speaking: Making Introductions

We make introductions in English in different occasions and contexts like when [illegible] someone for the first time or when we introduce someone to somebody. Introductions are formal or informal. Formal introductions often take place among colleagues in formal situations and titles should be used (e.g. Dr., Prof., Mr., Miss.,Mrs.Ato, W/ro,etc).On the other hand informal introductions are done among friends, or with people who know each other very well and call each other by name(Amaliraj. 2008).

What expressions do you use if:

You want to (have to) introduce yourself to other people?

You want to (have to) introduce a person to other person(s)?

- **Formal introductions**
- **Self introduction**

Look at the following dialogues.

Abebe: Hello, I am Abebe.

Martha: Hello, I am Martha.

Abebe: Nice to meet you.

Martha: Nice to meet you too.

Expressions for introducing oneself

- My name is…
- I'm …
- Allow me to introduce myself, my name is/I'm …
- (Hello,) Let me introduce myself, my name is/I'm…
- May I introduce myself, my name is/I'm…

Expressions for introducing a person

- **Formal**

A. Person to person

- Name I would like to introduce you to name…
- E.g., Bekele, I would like to introduce you to deputy inspector Maru. He is head of Department of Crime Protection of Region 14.
- Name, May I introduce you to name….
- E.g., Miss Martha, may I introduce you toMehiret. She is chief of Research and Community Service in ACA.
- name, I'd like you to meet name
- E.g., Inspector Markos, I'd like you to meet Mr. Tamirat. He works for Commercial Bank of Ethiopia.
- We usually use openers when someone introduces us with a person or persons.

Openers	Responses
How do you do?	How do you do?
It is a pleasure to meet you.	(Thank you) It's a pleasure to meet you, too.
Nice to meet you.	(Thank you) Nice to meet you, too.
I'm glad to meet you.	(Thank you) I'm glad to meet you, too.

lecturing is one of the oldest teaching approaches thus, proponents of lecture methods argue that it is an important teaching approach bic of the following reasons first it is quite economically it can be easily adopted. However, in my opinion there arguments are unsound bic of the following reasons 1st it makes learners passive recipient of knowledge from the lecture 2nd it does not accomodate individual differences further it gives superficial & transient knowledge. in general lecture method should not be used this case where there are many act teaching methods

- ✓ The set of positive integers less than 100 can be denoted by $\{1, 2, 3, \ldots 99\}$.
- ✓ The set natural numbers can be denoted by $\mathbb{N} = \{1, 2, 3, \ldots\}$.

ii. Verbal method.

A set can be describe by words.

Example:

- ✓ The set of natural numbers less than 10.
- ✓ The set of integers.

iii. Set builder method

Another way to describe a set is to use **set builder** notation. We characterize all those elements in the set by stating the property or properties they must have to be members.

Example:

- ✓ $\mathbf{E} = \{x | x \text{ is an even integer and } 1 < x < 9\}$
- ✓ $\mathbb{N} = \{x | x \text{ is a natural number}\}$.
- ✓ $\mathbb{R} = \{x | x \text{ is a real number}\}$.

We will now introduce notation used to describe elements in sets. We write

i. $x \in A$ to denote that x is an element of the set A.

ii. $x \notin A$ to denote that 'x is not an element of the set A'

Illustrative Example

2. Write the following sets using set builder notation

a. $A = \{31, 32, 33, \ldots 298, 299\}$.

b. $B = \{2, 4, 8, 10, \ldots\}$.

c. $X = \{1, 3, 5, 7, \ldots\}$.

d. M = the set of whole numbers less than 100.

Solution:

a. $A = \{x | x \text{ is a natural number greater than 30 and less than 300.}\}$ or

$A = \{x | 30 < x < 300, x \in \mathbb{N}\}$.

b. $B = \{x | x \text{ is an even natural number}\}$. Or

$B = \{x | x = 2n, n \in \mathbb{N}\}$.

c. $X = \{x | x = 2n - 1, n \in \mathbb{N}\}$ or $\{2n - 1 | n \in \mathbb{N}\}$

d. $M = \{x | x \in W \text{ and } x < 100\}$.

3.3 Empty set, Finite and In finite set

3.3.1. Empty set

A set that contains no elements is called an **empty set** or **null set**. The empty set is denoted by the symbol ϕ or $\{\}$

- ✓ The cardinal number of empty set is **0**.

Example of empty set:

$A = \{x | x \in \mathbb{N}$ and $1 < x < 2\}$ is empty set and $n(A) = 0$

$B = \{x | x \in \mathbb{R}$ and $x^2 = -1\}$ is empty set and $n(B) = 0$.

- ✓ The **cardinal number** of a set A, denoted by **n(A)**, indicates the number of element in the set A.

3.3.2. Finite set and Infinite set

i. A set is a **finite set** if the number of elements of the set is zero or a natural number, n.

ii. A set is said to be **infinite** if it is not finite.

Illustrative Example

3. What is the cardinality of each of these sets?

a. $A = \{a\}$ b. $A = \emptyset$ c. $A = \{\emptyset\}$ d. $A = \{\emptyset, \{\emptyset, \}\emptyset\}\}$

e. $A = \{\emptyset, \}a\}, \{\emptyset, a\}\}$ f. $A = \{x | x \in \mathbb{N}$ and $1 < x < 2\}$

Solution:

You know that 'cardinal number of a set A' is number of elements in the set A, denoted by n(A).

a. $n(A) = 1$

b. $n(A) = 0$

c. $n(A) = 1$

d. $n(A) = 2$

e. $n(A) = 3$

f. $n(A) = 0$

4. Determine which of the following set are finite or infinite.

a. A = {seasons in the year}.

b. A = {positive integers less than 1}.

c. A = {odd integers}

d. A = {positive factor of 24}.

e. A = the set of lines parallel to the y – axis.

f. A = The set of animals living on earth.

g. $A = \{x | x^2 = -1$ and $x \in \mathbb{R}\}$.

h. $A = \{x | 0 < x < 1$ and $x \in Q\}$.

Solution:

a. A is **finite** since there are four seasons in the year, i.e $n(A) = 4$
b. A is **finite**, since there is no positive integer less than 1, hence A is empty set. Thus empty set is finite set and $n(A) = 0$.
c. A is **infinite**
d. A is **finite**, because factor of 24 are 1, 2, 3, 4, 6, 8, 12 and 24. And $n(A) = 8$.
e. A is **infinite**
f. A is **finite**
g. A is **finite** because there is no real number whose square is negative, hence A is empty set Thus, $n(A) = 0$.
h. A is **infinite.**

3.4 Equal set and Equivalent set

3.4.1 Equal set

Set A is **equal** to set B, denoted by $A = B$, if and only if A and B have exactly the same elements.

Example:
Let $A = \{a, b, c\}$, $B = \{b, c, d\}$, and $C = \{c, a, b\}$ Then $A \neq B$, however $A = C$

3.4.2. Equivalent Sets

Set A is **equivalent** to set B, denoted by $A \leftrightarrow B$ (or $A \sim B$) if and only if A and B have the same number of elements, i.e $n(A) = n(B)$

Example: Let $A = \{a, b, c\}$, and $B = \{1, 2, 3\}$
$n(A) = 3$, and $n(B) = 3$, so that $A \leftrightarrow B$

Illustrative Example

5. Which one of the following pair of sets are equal. **Justify.**
   a. $\{2, 3, 4\}$ and $\{2, 3, 4, 3\}$
   b. $\{\Gamma, \{2,3\}\}$ and $\{1, 2, 3\}$
   c. $A = \{x \mid x \in \mathbb{N}, x^2 = 3\}$ and $B = \{x \mid x \in \mathbb{R}, x^2 = -2\}$.
   d. $A = \{x \mid x \in \mathbb{R}, x^2 - x = 0\}$ and $B = \{x \mid x \in \mathbb{R}, x^2 - x^4 = 0\}$

Solution:

a. Order and repetition do not change a set, so that they are **equal** set.
b. The first set has two elements and the second set has three elements, so the sets are **not equal** and **non-equivalent**

c. $A = \{x | x \in \mathbb{N}, x^2 = 3\}$ there is no natural number x that satisfies $x^2 = 3$.
So, $\mathbf{A = \emptyset}$.
✓ $B = \{x | x \in \mathbb{R}, x^2 = -2\}$ there is no real number x that makes $x^2 = -2$, so that $\mathbf{B = \emptyset}$. That is, if A and B are both empty set, **then set A is equal to set B.**

d. First solve for x in both set
✓ $x^3 - x = 0 \Rightarrow x(x^2 - 1) = 0 \Rightarrow x(x - 1)(x + 1) = 0$
$\therefore A = \{0, 1, -1\}$.
✓ $x^2 - x^4 = 0 \Rightarrow x^2(1 - x^2) = 0 = \Rightarrow x^2(1 - x)(1 + x) = 0\}$
$\therefore B = \{0, 1, -1\}$.
$\therefore$ **Set A is equal to set B.**

6. If two sets are equal must they also be equivalent?
Answer: Yes. If the sets are equal then they have exactly the same elements, therefore they also have the same number of elements.

7. Which one of the following sets is equal to $A = \{x | x \in \mathbb{N}, -2 \leq x < 4\}$.
A. $\{-2, -1, 0, 1, 2, 3\}$ C. $\{1, 2, 3\}$
B. $\{1, 2, 3, 4\}$ D. $\{0, 1, 2, 3, 4\}$
Solution:
$A = \{x | x \in \mathbb{N}, -2 \leq x < 4\} = \{1, 2, 3\}$. **Answer: C**

8. List the set which is equal to $A = \{x | x \in \mathbb{N}, -1 \leq x \leq 4\}$.
Solution: $A = \{1, 2, 3, 4\}$.

Which one of the following sets is equivalent to $A = \{2, \{3,4\}\}$
a. $\{2, 3, 4\}$ b. $\{\{a, b\}\}$ c. $\{1, \emptyset\}$. d. $\{\emptyset, \{2, \emptyset\}\}$
Solution: given set $A = \{2, \{3,4\}\}$ has two elements and
a. $\{2, 3, 4\}$ has three elements, so that the set are **not equivalent** to set A
b. $\{\{a,b\}\}$ has one element, so that the set are **not equivalent** to set A.
c. $\{1, \emptyset\}$ has two elements, so that the two sets are **equivalent.**
d. $\{\emptyset, \{2,\emptyset\}$ has two elements, so that the two sets are **equivalent.**

10. Which one of the following sets is equivalent to $A = \{\{1, 2, 3\}\}$
A. $S = \emptyset$ B. $E = \{\emptyset\}$ C. $F = \{1, 2, 3\}$ D. $G = \{x | x < x\}$
Solution: Here, we have $n(A) = 1$
the number of elements of a given set A is one i.e $n(A) = 1$.
➢ Also, the number of elements in set:
✓ $S = \emptyset$ is zero, $\therefore n(S) = 0$
✓ $B = \{\emptyset\}$ is one $\therefore n(B) = 1$
✓ $F = \{1, 2, 3\}$ is three, $\therefore n(B) = 3$
✓ $G = \{x | x < x\}$ is zero, $\therefore n(G) = 0$
Answer: B

11. Which one of the following sets is equal to $A = \{x | x \in \mathbb{Z} \text{ and } x^2 < 5\}$

A. $\{-2, -1, 0, 1, 2\}$ C. $\{0, 1, 2\}$

B. $\{-1, 0, 1\}$ D. $\{1, 2, 3, 4\}$

Solution: Since x is an integer, so that $A = \{-1, 0, 1\}$. **Answer:** B

12. Determine whether the following sets are equivalent. Justify

a. $A = \{2, 4, 6, 8, \ldots, 1000\}$ and $B = \{3, 6, 9, 12, 15, \ldots 1500\}$

b. $C = \{1, 4, 7, 10, 13, \ldots 2998\}$ and $D = \{1, 2, 3, 4, \ldots 1000\}$

c. $E = \{0, 1, 2, 3, 4, \ldots\}$ and $F = \{1, 2, 3, 4, 5, \ldots\}$

d. $G = \{x | x = 2n, \text{ and } n \in \mathbb{N}\}$ and $H = \{x | x = 4n \text{ and } n \in \mathbb{N}\}$;

Solution;

a. **Yes,** because there are $A = \frac{1000}{2} = 500$ elements in set A and $B = \frac{1500}{3} = 500$ elements in set B, so each has 500 elements.

b. **Yes,** because each set C and D has 1000 elements.

c. **Yes,** to each elements n in the first set corresponds the element $n + 1$ in the second set.

d. **Yes,** to each element x is the first set corresponds the element $2x$ in the second set.

3.5 Subsets, proper subsets and Power set.

3.5.1. Subsets of a set

Set A is a **subset** of set B, denoted by $A \subseteq B$, if and only if every element of A is also an element of B.

- ✓ A set with **n** elements has 2^n subsets.
- ✓ If A is not subset of B, if at least one element A does not belong to B. We write $A \not\subseteq B$.

Note: For any set A:

i. $\emptyset \subseteq A$ ii. $A \subseteq A$.

3.5.2. Proper subsets of a set

Set A is a **proper subset** of set B, denoted by $A \subset B$, if and only $A \subseteq B$ and $A \neq B$.

3.5.3. The power set

For any set A, the power set of A is *the set of all subset of the set A*. the power set of A is denoted by P(A).

✓ For any set A, i) $\emptyset \in P(A)$, ii) $A \in P(A)$

Illustrative Example

13. List all the subset, proper subset and power set of {a, b, c}

Solution: Let A = {a, b, c}.

- ✓ Subsets with no elements: ∅
- ✓ Subsets with one elements: {a}, {b}, {c}
- ✓ Subsets with two elements: {a, b}, {a, c}, {b, c}
- ✓ Subsets with three elements: {a, b, c}

Therefore:

i. all subsets of A = {a, b, c} are
∅, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}.

ii. all the proper subsets A = {a, b, c} are
∅, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}.

iii. the power set of A = {a, b, c} are:
P(A) = {∅, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}}

14. List all the subsets, proper subset and power set of empty set.

Solution: Let A = ∅.

i. subsets of empty set is ∅.
ii. empty set has no proper subsets.
iii. The power set of empty set is {∅}. i.e P (A) = {∅}.

15. Let A = {∅}. Then list all subset, proper subset and power set of A.

Solution:

- Subsets with no elements: ∅
- Subsets with one elements: {∅}. **Therefore**
  - i. all subsets of A = {∅} are ∅, {∅}.
  - ii. proper subsets of A = {∅} is ∅.
  - iii. the power set of A = {∅} is P (A) = {∅,{∅}}

16. Lets A = {∅, {0, 1}}. Then list all Subsets, power sets, and proper subsets of A.

Solution: Here at A = {∅, {0, 1}}.

- ✓ Subsets with no element: ∅
- ✓ Subsets with one elements: {∅}, {{0, 1}}.
- ✓ Subsets with two element: {∅, {0, 1}} Thus

i. all subset of A are ∅, {∅}, {{0, 1}} and {∅, {0, 1}}.
ii. the power set of A = {∅, {0, 1}}is given by
P(A) = {∅,{∅}, {{0, 1}}, {∅, {0, 1}}

17. Is $\emptyset \subset A$, for any set A?
Answer: **No,** let $A = \emptyset$, since $A \subset B$ if and only if $A \subseteq B$ and $A \neq B$.

18. Is $A \subseteq P(A)$, for any set A? **Answer**: No, $A \in P(A)$.

19. Let $A = \{\emptyset, \{1, \emptyset\}\}$. Determine whether these statements are true or false.
a. $\emptyset \in A$
b. $\{\emptyset\} \subseteq A$
c. $\{1, \emptyset\} \subseteq A$
d. $\{\{1, \emptyset\}\} \subseteq A$.
e. $\{\{1, \emptyset\}\} \in P(A)$.
f. $\{1\} \in P(A)$.

Solution: Elements of A are $\emptyset$, $\{1, \emptyset\}$
- ✓ Subsets of are $\emptyset$, $\{\emptyset\}$, $\{\{1, \emptyset\}\}$, $\{\emptyset, \{1, \emptyset\}\}$.
- ✓ Power set of A are $P(A) = \{\emptyset, \{\emptyset\}, \{\{1, \emptyset\}\}, \{\emptyset, \{1, \emptyset\}\}\}$.

Therefore, a. True c. False e. True
b. True d. True f. False

20. Find the number of subset, and proper subset of the following set.
a. $M = \{x, y, z\}$ b. $E = \{x, \{y, z\}\}$ c. $G = \{\{x, y, z\}\}$

Solution:
- ✓ A set with n– elements has 2^n subsets
- ✓ A set with n – elements has $2^n - 1$ proper subsets.

a. $\{x, y, z\}$ has three elements. It has $2^3 = 8$ subsets and $2^3 - 1 = 8 - 1 = 7$ proper subsets.
b. $E = \{x, \{y, z\}\}$ has two elements. It has $2^2 = 4$ subsets and $2^2 - 1 = 3$ proper subsets
c. $G = \{\{x, y, z\}\}$ has one elements. It has $2^1 = 2$ subsets and $2^1 - 1 = 1$ proper subsets

21. Determine whether each of these statements is true of false .
a. $\emptyset \in \{\emptyset\}$
b. $\{\emptyset\} \in \{\emptyset\}$
c. $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$
d. $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$
e. $\{3\} \subseteq \{\{3\}\}$
f. $\{2\} \subseteq \{\{2, 3\}\}$

Solution:
a. True c. True e. False
b. False d. True f. False

22. Let $A = \{2, 4, 6, 8, 10, \ldots\}$, $B = \{x \mid x = 2n + 2 \text{ and } n \in W\}$
$C = \{x \mid x = 4n \text{ and } n \in \mathbb{N}\}$. Then;
a. Which of the following sets are equal?
b. Which sets are proper subsets of the other sets?

Solution:
a. Since n is whole number, i.e, $n = 0, 1, 2, \ldots$ so that $B = \{2, 4, 6, 8, \ldots\}$, $\therefore A = B$
b. Since $n \in \mathbb{N}$, that is $n = 1, 2, 3, \ldots$, so that $C = \{4, 8, 12, 16, \ldots\}$. **Therefore, $C \subset A$ and $C \subset B$**

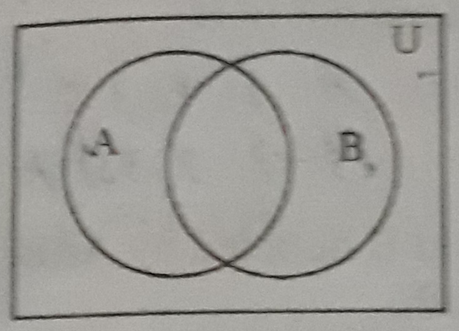
3.6 Venn Diagram, Universal set

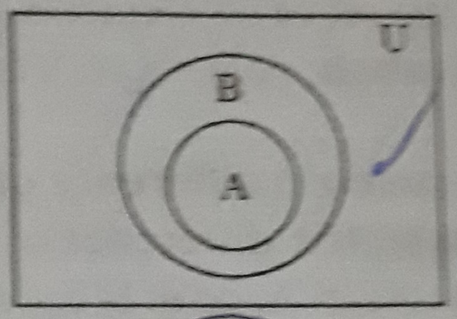
i. **Venn diagrams** are a pictorial representation of the inter – relationship among two or more sets. represented

ii. A **universal set**, demoted by U, is the set that contains all elements under investigation usually belong to some fixed large set.

In the Venn diagrams:

- ✓ The universal set (U) is represented by a rectangle, and other sets are represented by circle lying within rectangle





$A \subseteq B$

3.7 Set Operation

This section introduces, the basic operations of Union, intersection complement of a set.

3.7.1. Union and Intersection

i. The **Union** of two sets A and B, denoted by **A∪B**, is the set of all elements in A or in B

Symbols: $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

ii. The **intersection** of two sets A and B denoted by **A∩B**, is the set of all elements common to both A and B

Symbols: $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$.

3.7.2. Complements and symmetric Difference of set.

i. The complement of A set.

The **complement** (or absolute complement) of a set A, denoted by A' is the set of all elements in the universal set U that are not in A.

Symbols: $A' = \{x \mid x \in U \text{ and } x \notin A\}$ or

$A' = U - A = U \backslash A = \{x \mid x \in U \text{ and } x \notin A\}$.

ii. The relative complement of set.

The **relative complement** of set B with respect to a set A, denoted by **A\B** is the set of all elements in A that are not in B.

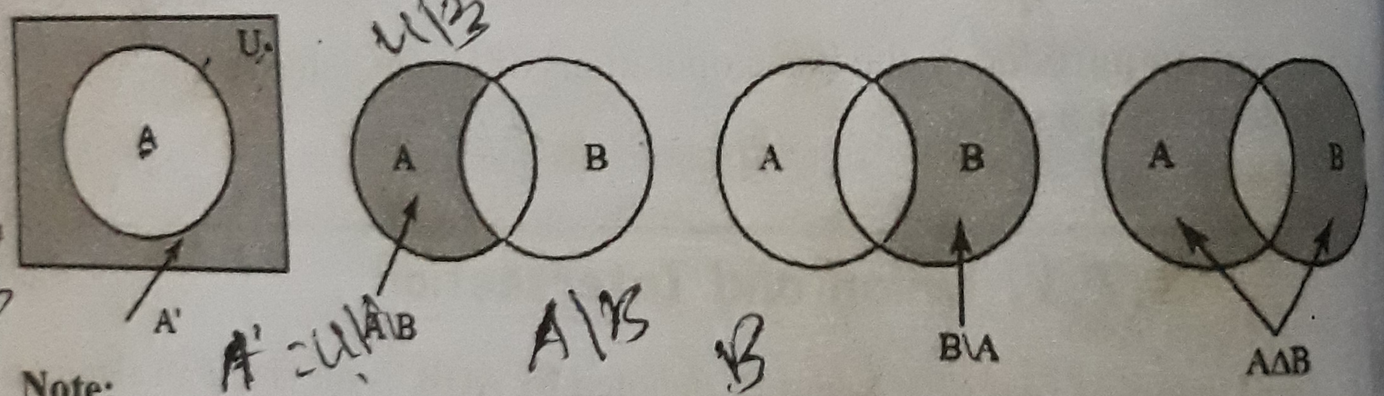
Symbols: $A \backslash B = \{x \in A \text{ and } x \notin B\}$.

iii. The symmetric difference of set.

The **symmetric difference** of set A and B denoted by **AΔB,** is the set of all elements in A, or in B but not to both.

Symbols: $A \Delta B = (A \backslash B) \cup (B \backslash A) = (A \cup B) \backslash (A \cap B)$.

In Venn – diagrams, A', A\B, B\A and AΔB can be shown in shaded, below.



Note:

- A\B ⟶ only A, but no B.
- B\A ⟶ only B, but no A.

Illustrative Example

23. Let U = {a, b, c, d, e}, A = {a, b, c} B = {b, c}, C = {a, d}.

Find

a. A ∪ B
b. A ∩ C
c. A ∩ B
d. B ∩ C
e. A'
f. B'
g. C'
h. A\B
i. B\A
j. B\C
k. C\B
l. AΔB.

Solution:

a. $A \cup B = \{a, b, c\} \cup \{b, c\} = \{a, b, c\}$

b. $A \cap C = \{a, b, c\} \cap \{a, d\} = \{a\}$

3.7.2. Complements and symmetric Difference of set.

i. The complement of A set.

The **complement** (or absolute complement) of a set A, denoted by A' is the set of all elements in the universal set U that are not in A.

Symbols: $A' = \{x | x \in U \text{ and } x \notin A\}$ or

$A' = U - A = U \backslash A = \{x | x \in U \text{ and } x \notin A\}$.

ii. The relative complement of set.

The **relative complement** of set B with respect to a set A, denoted by **A\B** is the set of all elements in A that are not in B.

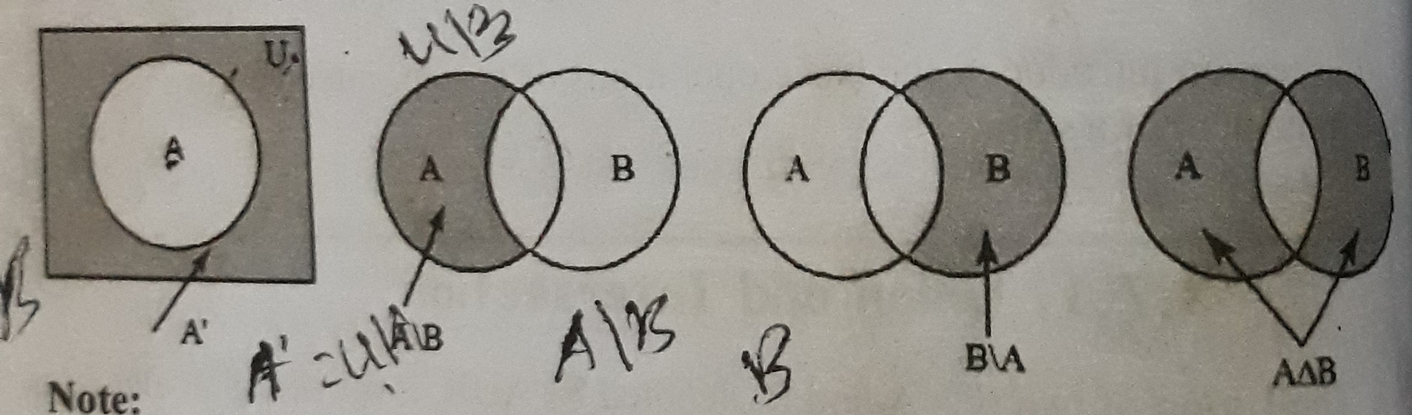
Symbols: $A \backslash B = \{x \in A \text{ and } x \notin B\}$.

iii. The symmetric difference of set.

The **symmetric difference** of set A and B denoted by **AΔB,** is the set of all elements in A, or in B but not to both.

Symbols: $A \Delta B = (A \backslash B) \cup (B \backslash A) = (A \cup B) \backslash (A \cap B)$.

In Venn – diagrams, A', A\B, B\A and AΔB can be shown in shaded, below.



Note:

- A\B ⟶ only A, but no B.
- B\A ⟶ only B, but no A.

Illustrative Example

23. Let U = {a, b, c, d, e}, A = {a, b, c} B = {b, c}, C = {a, d}.

Find

a. A ∪ B
b. A ∩ C
c. A ∩ B
d. B ∩ C
e. A'
f. B'
g. C'
h. A\B
i. B\A
j. B\C
k. C\B
l. AΔB.

Solution:

a. $A \cup B = \{a, b, c\} \cup \{b, c\} = \{a, b, c\}$

b. $A \cap C = \{a, b, c\} \cap \{a, d\} = \{a\}$

c. $A \cap B = \{a, b, c\} \cap \{b, c\} = \{b, c\}$
d. $B \cap C = \{b, c\} \cap \{a, d\} = \emptyset$
e. $A' = U \setminus A = U - A = \{d, e\}$
f. $B' = U \setminus B = U - B = \{a, d\}$
g. $C' = U \setminus C = U - C = \{b, c, e\}$.
h. $A \setminus B = A - B = \{a\}$ ⟵ $A \cap B'$
i. $B \setminus A = B - A = \emptyset$ ⟵ $B \cap A'$
j. $B \setminus C = B - C = \{b, c\}$ ⟵ $B \cap C'$
k. $C \setminus B = C - B = \{a, d\}$ ⟵ $C \cap B'$
l. $A \Delta B = (A \setminus B) \cup (B \setminus A) = \{a\} \cup \emptyset = \{a\}$.

24. Let $A = \{a, \{b, c\}\}$, and $B = \{a, b, c\}$. Then find:
a. The relative complement of A with respect to B.
b. The complement of B relative to A
c. $A \Delta B$ d. $A \cup B$ e. $A \cap B$

Solution:

a. $B \setminus A = B - A = \{b, c\}$,
b. $A \setminus B = A - B = \{\{b, c\}\}$.
c. $A \Delta B = (A \setminus B) \cup (B \setminus A) = \{b, c, \{b, c\}\}$.
d. $A \cup B = \{a, \{b, c\}, b, c\}$.
e. $A \cap B = \{a\}$.

25. Let $A \cap B' = \{1, 2, 3\}$ and $A' \cap B = \{0, 4\}$.
Then which of the following is equal to $A \Delta B$?
A. $\emptyset$ B. $\{1, 2, 3\}$ C. $\{0, 4\}$ D. $\{0, 1, 2, 3, 4\}$

Solution: Here at, $A \cap B' = A \setminus B$ and $A' \cap B = B \setminus A$
$\therefore A \Delta B = (A \setminus B) \cup (B \setminus A) = \{0, 1, 2, 3, 4\}$ **Answer: D**

26. Let $A \cap B = \{1, 2, 3\}$, $A \cup B = \{1, 2, 3, 4, 5\}$ then $A \Delta B$ is equal to:
A. $\{3, 4, 5\}$ B. $\{3, 4\}$ C. $\{1, 2, 3, 4, 5\}$ D. $\emptyset$

Solution: $A \Delta B = (A \cup B) \setminus (A \cap B) = \{3, 4\}$ **Answer: B**

27. Let $A = \{-2, \emptyset, \{0, 1\}\}$.
a. How many subsets does A have?
b. How many proper subsets does A have?

Solution: Here at, $n(A) = 3$. Then
a. Set A have $2^n = 2^3 = 8$ subsets.
b. Set A have $2^n - 1 = 2^3 - 1 = 7$ proper subsets.

3.8 Set properties (Identities)

1. $A \cup (B \cup C) = (A \cup B) \cup C$ ⟵ *Associative law.*
2. $A \cap (B \cap C) = (A \cap B) \cap C$ ⟵ *Associative law.*
3. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ ⟵ *Distributive law.*
4. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ ⟵ *Distributive law.*
5. $A \cup \emptyset = A$ and $A \cap U = A$ ⟵ *Identity law.*
6. $A \cup U = U$ and $A \cap \emptyset = \emptyset$ ⟵ *Identity law.*
7. $A \cap A' = \emptyset$ and $A \cup A' = U$ ⟵ *Complement law.*
8. $U' = \emptyset$ and $\emptyset' = U$ ⟵ *Complement law.*
9. $(A \cup B)' = A' \cap B'$ ⟵ *Demorgan's law.*
10. $(A \cap B)' = A' \cup B'$ ⟵ *Demorgan's law.*

Illustrative Example

28. Let $A \cap B = \{-1, 0, 1\}$ and $B \cap C = \{2, -1, 0\}$ Then $B \cap (A \cup C)$
A. $\{0, -1\}$ B. $\{1, 2\}$ C. $\{-1, 0, 1, 2\}$ D. $\{-1, 0, 2\}$
Solution: Refer # 3.8.

$$B \cap (A \cup C) = (B \cap A) \cup (B \cap C)$$
$$= \{-1, 0, 1\} \cup \{2, -1, 0\} = \{-1, 0, 1, 2\}$$

29. Let $A \cup B = \{1, 2, 3\}$ and $A \cup C = \{3, 4, 5\}$. Then $A \cup (B \cap C)$ **Answer: C**
A. $\{1, 2, 3, 4, 5\}$ B. $\{3\}$ C. $\{1, 2, 4, 5\}$ D. $\{3, 4, 5\}$
Solution: Refer # 3.8.

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C) = \{3\}.$$

30. Let A and B be two sets. If $A \setminus B = \emptyset$, then which of the following is always true. **Answer: B**
A. $B \setminus A = A \setminus B$ C. $B \setminus A = \emptyset$
B. $A \cup B = B$ D. $A \cap B = B - A$
Solution: Here at, $A \setminus B = \emptyset$

$A \cup B = B$ if $A \subseteq B$ and $A \neq B$.

31. For any sets A and B, which of the following is not true? **Answer: B**
A. $(A \setminus B) \cup (A \cap B) = A$ C. $A \cup (B \setminus A) = A$
B. $A \setminus (A \cap B) = A \setminus B$ D. $(A \setminus B) \cup (B \setminus A) = (A \cup B) \setminus (A \cap B)$
Solution:

Case 1. If $A \cap B = \emptyset$, then $(A \setminus B) \cup \emptyset = A \setminus B = A$

Case 2. If $A \subseteq B$, then $(A \setminus B) \cup (A \cap B) = \emptyset \cup (A \cap B) = A$

For any sets A and B

i. $A\setminus(A\cap B) = A\cap(A\cap B)' = A\cap(A'\cup B') = (A\cap A')\cup(A\cap B')$
$\emptyset\cup(A\cap B') = A\cap B' = A\setminus B$

ii. $A\cup(B\setminus A) = A\cup(B\cap A') = (A\cup B)\cap(A\cup A') = (A\cup B)\cap U = A\cup B.$

Answer: C

32. If $(A\setminus B)\cup B = A$, then which of the following is always true?
A. $B=\emptyset$ B. $A=B$ C. $B\subseteq A$ D. $A\subseteq B$

Solution: $(A\setminus B)\cup B = A\cup B = A$ if $B\subseteq A$ **Answer: C**

33. For any two sets A and B $(A'\cup B)'$ is equal to:
A. B B. $A\setminus B$ C. $B\setminus A$ D. A

Solution: $(A'\cup B)' = A\cap B' = A\setminus B$ **Answer B**

34. Simplify:
a. $A'\cap(A\cup B)$ b. $A\cap(A'\cup B)$
c. $A'\cup(A\cap B)$ d. $A'\cup(A\cup B)$.

Solution:

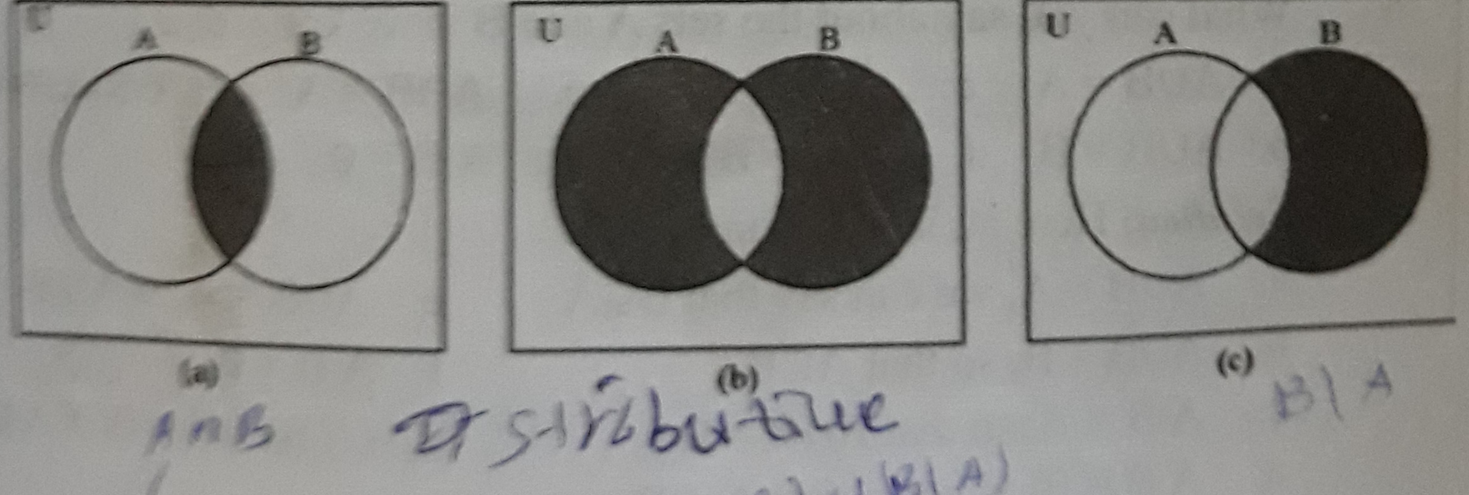
a. $A'\cap(A\cup B) = (A'\cap A)\cup(A'\cap B) = (A\setminus A)\cup(A'\cap B) = \emptyset\cup(A'\cap B)$
$= \emptyset\cup(B\setminus A) = B\setminus A.$

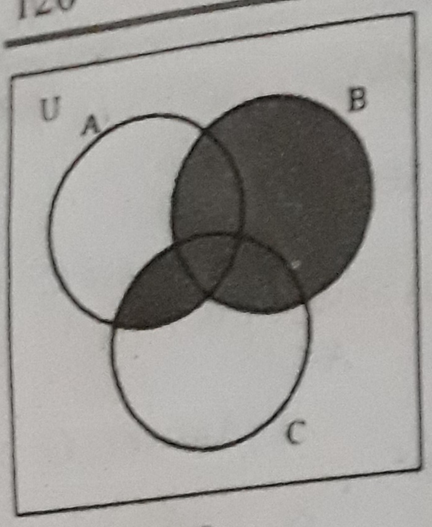
b. $A\cap(A'\cup B) = (A\cap A')\cup(A\cap B) = \emptyset\cup(A\cap B) = A\cap B$

c. $A'\cup(A\cap B) = (A'\cup A)\cap(A'\cup B)$
$= (U)\cap(A'\cup B)$
$= U\cap(A\cap B')'$
$= \cup\cap(A\setminus B)'$
$\therefore A\cup(A\cap B) = (A\setminus B)' = (A\cap B')' = A'\cup B.$

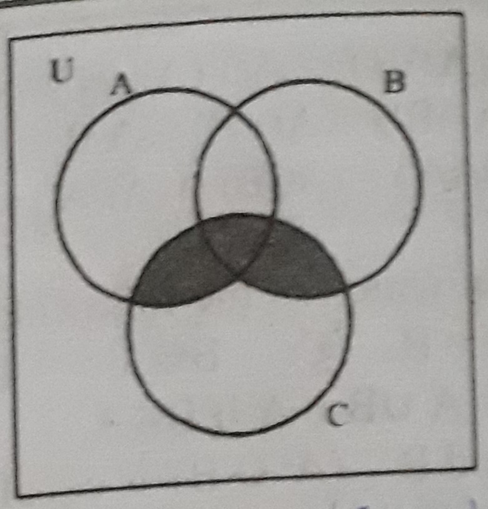
d. $A'\cup(A\cup B) = (A'\cup A)\cup B = U\cup B = U$

35. In each of the following Venn- diagrams, use the set notation to describe the shaded region.

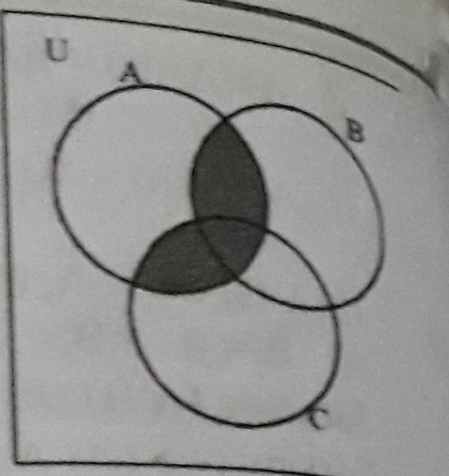




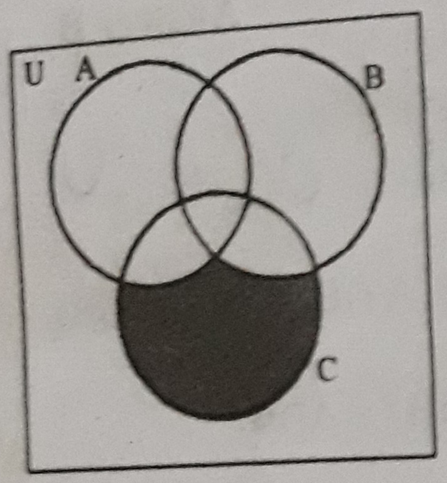
(d)



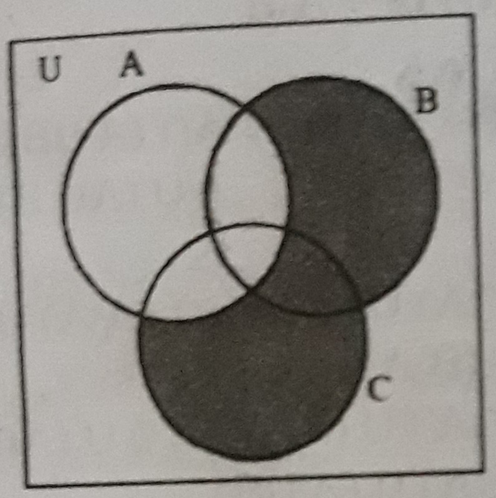
(e)



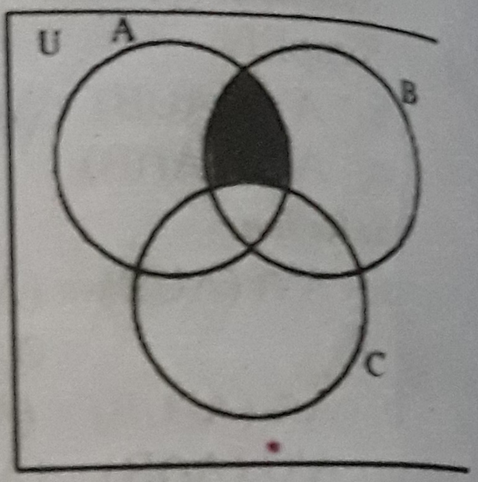
(f)



(g)



(h)



(i)

Solution:

a. $A \cap B$ b. $A \Delta B = (A \setminus B) \cup (B \setminus A)$ c. $B \setminus A$

d. $B \cup (A \cap C)$ e. $(A \cup B) \cap C$ f. $(B \cup C) \cap A$

g. $(A \cup B)' \cap C$ or $C \setminus (A \cup B)$. h. $(B \cup C) \setminus A$ or $(B \cup C) \cap A'$

i. $(A \cap B) \setminus C$

36. What can you say about the sets A and B if we know that

a. $A \cup B = A$ c. $A \setminus B = A$ e. $A \cap B = A$ g. $(A \setminus B) \cup B = B$

b. $A \cup B = B$ d. $A \setminus B = B \setminus A$ f. $A \setminus B = \emptyset$.

Solution: Here at, in set A and set B.

a. $A \cup B = A$, we can say that $B \subseteq A$.

b. $A \cup B = B$, so that $A \subseteq B$.

c. $A \setminus B = A$, so that $A \cap B = \emptyset$

d. $A \setminus B = B \setminus A$, whenever $A = B$

e. $A \cap B = A$, if $A \subseteq B$.

f. $A \setminus B = \emptyset$, if $A \subseteq B$.

g. $(A \setminus B) \cup B = B$ if $A \subseteq B$

37. Determine whether each of the following is true or false.

a. If $A \subseteq B$, then $B' \subseteq A'$, for any set A and B

b. If $x \in (B \setminus A)$, then $x \in B$ and $x \notin A$.

c. $(A \setminus C) \cap (C \setminus B) = \emptyset$, for any set A, B and C.

d. $A \cup (B \cap C) = (A \cup B) \cap C$ for all Sets A, B, and C

e. $A \setminus B = B' \setminus A'$ for any sets A and B
f. $A \setminus B = A' \setminus B'$ for any set A and B.
g. $A' \cap (C \setminus B) = C \setminus (A \cup B)$ for any set A, B, and C
h. $A' \cap (B \cup C) = (B \cup C) \setminus A$, for any set A, B, and C.
i. $A \cap (B \cup C) = (A \cap B) \cup C$ for all set A, B, and C.

Solution:

a. true b. true c. true
d. false (refer # 34 figure (d) and (e)
e. true. Since $B' \setminus A' = B' \cap A = A \cap B' = A \setminus B$.
f. false, because $A' \setminus B' = A' \cap B = B \cap A' = B \setminus A$.
g. true, since $C \setminus (A \cup B) = C \cap (A \cup B)' = C \cap (A' \cap B')$
$= A' \cap (C \cap B') = A' \cap (C \setminus B)$.
h. true, since $(B \cup C) \setminus A = (B \cup C) \cap A' = A' \cap (B \cup C)$
i. false

3.8.1. Applications of Sets.

Suppose A, and B are finite sets.

i. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.
If $A \cap B = \emptyset$, then $n(A \cup B) = n(A) + n(B)$.
ii. $n(A \setminus B) = n(A) - n(A \cap B)$.
iii. $n(A') = n(U) - n(A)$.
iv. $n(A \Delta B) = n(A \setminus B) + n(B \setminus A) = n(A \cup B) - n(A \cap B)$
v. $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

Illustrative Example

38. Let $n(A) = 22$, $n(B) = 18$, and $n(A \cup B) = 33$
Find a. $n(A \cap B)$ c. $n(A \setminus B)$
b. $n(A \Delta B)$ d. $n(B \setminus A)$

Solution: Refer # 3.8.1.

a: $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.
$33 = 22 + 18 - n(A \cap B)$
$\therefore n(A \cap B) = 40 - 33 = 7$
b. $n(A \Delta B) = n(A \cup B) - n(A \cap B) = 33 - 7 = 26$
c. $n(A \setminus B) = n(A) - n(A \cap B) = 22 - 7 = 16$
d. $n(B \setminus A) = n(B) - n(A \cap B) = 18 - 7 = 11$.

39. Let $n(A \setminus B) = 12$ and $n(B \setminus A) = 15$ and $n(A \cap B) = 10$
Find a. $n(A)$, and $n(B)$ b. $n(A \Delta B)$

Solution: Refer # 3.8.1.

a. $n(A\backslash B) = n(A) - n(A \cap B)$ and $n(B\backslash A) = n(B) - n(A \cap B)$

$\Rightarrow 12 = n(A) - 10$ and $15 = n(B) - 10$

$\therefore$ **$n(A) = 22$** and **$n(B) = 25$**

b. $n(A \Delta B) = n(B\backslash A) + n(A\backslash B) = 12 + 15 = 27$

40. Let $U = 30$, $n(A) = 12$, $n(B) = 13$ and $n(A \cup B) = 20$.

Find a. $n(A')$ d. $n(A' \cup B')$ g. $n(A\backslash B)'$

b. $n(B')$ e. $n(A \Delta B)$ h. $n(A'\backslash B')$

c. $n(A' \cap B')$ f. $n(A' \cup B)$ i. $n(B'\backslash A')$

Solution: Here at: $n(A \cap B) = n(A) + n(B) - n(A \cup B)$

a. $n(A') = n(U) - n(A) = 30 - 12 = 8$.

b. $n(B') = n(U) - n(B) = 30 - 13 = 17$.

c. Since $A' \cap B' = (A \cup B)'$, so that:

$n(A' \cap B') = n(A \cup B)' = n(U) - n(A \cup B) = 30 - 20 = \mathbf{10}$

d. We know that $A' \cup B' = (A \cap B)'$

$\therefore n(A' \cup B') = n(A \cap B)' = n(U) - n(A \cap B) = 30 - 5 = \mathbf{25}$.

e. $n(A \Delta B) = n(A \cup B) - n(A \cap B) = 20 - 5 = 15$.

f. $A' \cup B = (A \cap B')' = (A\backslash B)'$

$\therefore n(A \cup B') = n(A\backslash B)' = n(U) - n(A\backslash B) = 30 - (12 - 5) = 23$

g. $n(A\backslash B)' = n(U) - n(A\backslash B) = 30 - (12 - 5) = \mathbf{23}$.

h. we know that $A'\backslash B' = A' \cap B = B\backslash A$.

$\therefore n(A'\backslash B') = n(B\backslash A) = n(B) - n(A \cap B) = 13 - 5 = 8$

i. we know that $B'\backslash A' = B' \cap A = A\backslash B$.

$\therefore n(B'\backslash A') = n(A\backslash B) = n(A) - n(A \cap B) = 12 - 5 = 7$

41. Let $n(A) = 16$ and $n(B) = 12$. **Find**

a. maximum and minimum possible number of elements in $A \cup B$.

b. maximum and minimum possible number of elements in $A \cap B$.

Solution: Refer # 3.8.1.$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

a. $n(A \cup B)$ is maximum if $A \cap B = \emptyset$, i.e. $n(A \cap B) = 0$

$\therefore$ the maximum number of elements in $A \cup B$ is $n(A \cup B) = n(A) + n(B) = 16 + 12 = 28$.

$n(A \cup B)$ is least if $A \cap B = B$ and $B < A$.

$\therefore$ Least number of $A \cup B$: $n(A \cup B) = n(A) + n(B) - n(A \cap B) = 16 + 12 - 12 = 16$

$\therefore 16 \leq n(A \cup B) \leq 28$.

b. Maximum possible number of elements in $A \cap B$ is $n(A \cap B) = n(B) = 12$

Least possible number of elements in $A \cap B$ is $n(A \cap B) = 0$, if $A \cap B = \emptyset$

42. In a school of 150 students, 115 takes Mathematics and 85 take Physics. Each student takes at least one of these subjects.

a. How many take both subjects?

Solution: Refer # 3.8.1.

a. $n(A \setminus B) = n(A) - n(A \cap B)$ and $n(B \setminus A) = n(B) - n(A \cap B)$

$\Rightarrow 12 = n(A) - 10$ and $15 = n(B) - 10$

$\therefore$ **$n(A) = 22$** and **$n(B) = 25$**

b. $n(A \Delta B) = n(B \setminus A) + n(A \setminus B) = 12 + 15 = 27$

40. Let $U = 30$, $n(A) = 12$, $n(B) = 13$ and $n(A \cup B) = 20$.

Find a. $n(A')$ d. $n(A' \cup B')$ g. $n(A \setminus B)'$

b. $n(B')$ e. $n(A \Delta B)$ h. $n(A' \setminus B')$

c. $n(A' \cap B')$ f. $n(A' \cup B)$ i. $n(B' \setminus A')$

Solution: Here at: $n(A \cap B) = n(A) + n(B) - n(A \cup B)$

a. $n(A') = n(U) - n(A) = 30 - 12 = 8$.

b. $n(B') = n(U) - n(B) = 30 - 13 = 17$.

c. Since $A' \cap B' = (A \cup B)'$, so that:

$n(A' \cap B') = n(A \cup B)' = n(U) - n(A \cup B) = 30 - 20 = \mathbf{10}$

d. We know that $A' \cup B' = (A \cap B)'$

$\therefore n(A' \cup B') = n(A \cap B)' = n(U) - n(A \cap B) = 30 - 5 = \mathbf{25}$.

e. $n(A \Delta B) = n(A \cup B) - n(A \cap B) = 20 - 5 = 15$.

f. $A' \cup B = (A \cap B')' = (A \setminus B)'$

$\therefore n(A \cup B') = n(A \setminus B)' = n(U) - n(A \setminus B) = 30 - (12 - 5) = 23$

g. $n(A \setminus B)' = n(U) - n(A \setminus B) = 30 - (12 - 5) = \mathbf{23}$.

h. we know that $A' \setminus B' = A' \cap B = B \setminus A$.

$\therefore n(A' \setminus B') = n(B \setminus A) = n(B) - n(A \cap B) = 13 - 5 = 8$

i. we know that $B' \setminus A' = B' \cap A = A \setminus B$.

$\therefore n(B' \setminus A') = n(A \setminus B) = n(A) - n(A \cap B) = 12 - 5 = 7$

41. Let $n(A) = 16$ and $n(B) = 12$. **Find**

a. maximum and minimum possible number of elements in $A \cup B$.

b. maximum and minimum possible number of elements in $A \cap B$.

Solution: Refer # 3.8.1.$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

a. $n(A \cup B)$ is maximum if $A \cap B = \emptyset$, i.e. $n(A \cap B) = 0$

$\therefore$ the maximum number of elements in $A \cup B$ is $n(A \cup B) = n(A) + n(B) = 16 + 12 = 28$.

$n(A \cup B)$ is least if $A \cap B = B$ and $B < A$.

$\therefore$ Least number of $A \cup B$: $n(A \cup B) = n(A) + n(B) - n(A \cap B) = 16 + 12 - 12 = 16$

$\therefore 16 \leq n(A \cup B) \leq 28$.

b. Maximum possible number of elements in $A \cap B$ is $n(A \cap B) = n(B) = 12$

Least possible number of elements in $A \cap B$ is $n(A \cap B) = 0$, if $A \cap B = \emptyset$

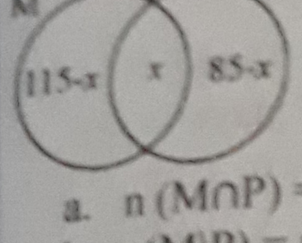
42. In a school of 150 students, 115 takes Mathematics and 85 take Physics. Each student takes at least one of these subjects.

a. How many take both subjects?

b. How many take Mathematics only?
c. How many take Physics only>

Solution:

- Let M = student who take math's and n (M) = 115
- Let P = student who take Physics and n (P) = 85



- Let X = M∩P = students who take both

$n(M \cup P) = n(M) + n(P) - n(M \cap P)$

$150 = 115 + 85 - n(M \cap P)$

a. $n(M \cap P) = 115 + 85 - 150 = 50$
b. $n(M \setminus P) = n(M) - n(M \cap P) = 115 - 50 = 65$ take Mathematics only.
c. $n(P \setminus M) = n(P) - n(M \cap P) = 85 - 50 = 35$ take Physics only

43. Among 110 students, 45 take Amharic 35 take Afaan Oromo language and 20 take both Amharic and Afaan Oromo.

Find the number of student who:

a. do not take Amharic language
b. do not take Afaan – Oromo language.
c. take Amharic or Afaan Oromo.
d. take Amharic, but not Afaan – Oromo
e. take Afaan – Orom, but not Amharic.
f. take exactly one of the two language.
g. take neither Amharic nor Afaan Oromo.

Solution: Let A = Amharic language

Let B = Afaan – Oromo.

a. $n(A') = n(U) - n(A) = 110 - 45 = 65$
b. $n(B') = n(U) - n(B) = 110 - 35 = 75$
c. $n(A \cup B) = n(A) + n(B) - n(A \cap B) = 45 + 35 - 20 = 60$
d. $n(A \setminus B) = n(A) - n(A \cap B) = 45 - 20 = 15$
e. $n(B \setminus A) = n(B) - n(A \cap B) = 35 - 20 = 15$
f. $n(A \Delta B) = n(A \setminus B) + n(B \setminus A) = 25 + 15 = 40.$
g. $n(A \cup B)' = n(U) - n(A \cup B) = 110 - 60 = 50.$

44. In a class of 33 students, 18 take English, 20 take Mathematics and 7 students take neither. How many take.

a. both subject
b. English only
c. Mathematics only
d. Exactly one of the two subjects.

Solution: Let E = take English and Let M = take Mathematics.

$n(E \cup M)' = n(U) - n(E \cup M)$ ⟵ neither of the subject.

$\Rightarrow 7 = 33 - n(E \cup M)$; therefore $n(E \cup M) = 33 - 7 = 26$

a. $n(E \cap M) = n(E) + n(M) - n(E \cup M) = 18 + 20 - 26 = 12$
b. $n(E \setminus M) = n(E) - n(E \cap M) = 18 - 12 = 6$

c. $n(M \setminus E) = n(M) - n(E \cap M) = 20 - 12 = 8$
d. $n(A \Delta B) = n(E \setminus M) + n(M \setminus E = 6 + 8 = 14)$

45. in a class of 50 students, 15 play football only, 10 play both football and basketball and 5 play neither football nor basketball. How many students play basketball?
A. 10 B. 20 C. 30 D. 25

Solution: Here at,
Let F = football player and B = Basketball player
$n(F \setminus B = 15$ and $n(F \cap B) = 10$ and $n(F \cup B)' = 5$. Then:

- ✓ $n(F \setminus B) = n(F) - n(F \cap B)$
  $15 = n(F) - 10$ so that $\mathbf{n(F) = 15 + 10 = 25.}$
- ✓ $n(F \cup B)' = n(U) - n(F \cup B)$
  $5 = 50 - n(F \cup B) \Rightarrow \mathbf{n(F \cup B) = 50 - 5 = 45}$
- ✓ $n(F \cup B) = n(F) + n(B) - n(F \cap B)$
  $45 = 25 + n(B) - 10$ so that: $\mathbf{n(B) = 45 - 15 = 30}$

Answer: C

3.9 Cartesian product of sets.

Let A and B be any two non-empty sets. The set of all ordered pairs (a,b) where $a \in A$ and $b \in B$ is called **the Cartesian product,** of A and B.

Symbols: $\mathbf{A \times B = \{(a, b)}$, where $\mathbf{a \in A, b \in B\}}$, Read as " A cross B"
Note: (a,b) = (c,d) if and only if a = c and b = d.
✓ Suppose A and B are finite. Then $n(A \times B) = n(A).\ n(B)$

Illustrative Example

46. Let A = {1, 2} and B = {x, y, z}. Then
Find a. $A \times B$ b. $B \times A$ c. $n(A \times B)$
Solution:

a. $A \times B = \{(1, x), (1, y), (1, z), (2, x), (2, y), (2, z)\}$
b. $B \times A = \{(x, 1), (y, 1), (z, 1), (x, 2), (y, 2), (z, 2)\}$
c. $n(A \times B) = n(A) \cdot n(B) = (2)(3) = 6.$

47. Let A = {a, b, c}. Then find $A \times A$.
Solution:
$A \times A = \{(a, a), (a, b), (a, c), (b, b), (b, c), (c, c), (c, b), (c, a), (b, a)\}$

48. Find x and y if
a. $(x + 4, 5) = (6, 4x - y)$ b. $(x - 2y, 11) = (-1, x + y)$
Solution: Here at, $(a, b) = (c, d) \Rightarrow a = c$ and $b = d$

a. $x + 4 = 6$ imply that $x = 6 - 4 = 2$
$4x - y = 5$ imply that $4(2) - y = 5$ and $y = 3$
$\therefore \mathbf{x = 2}$ **and** $\mathbf{y = 3.}$

b. $x-2y=-1$ and $x+y=11$

$$\begin{array}{r} \begin{cases} x-2y=-1 \\ x+y=11 \end{cases} \\ \hline \end{array}$$

$0-3y=-12$ and $y=\frac{12}{3}=4.$

To find for x, we put $y=4$ in $x+y=11$

$\therefore$ **$x=11-4=7$ and $y=4$.**

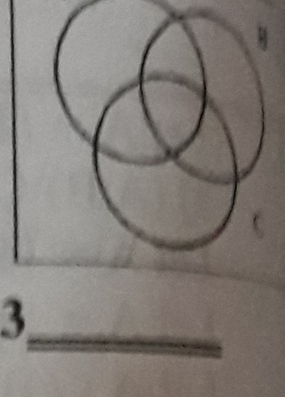
Review Exercise 3

1. Let $A=\{2,3,4,5\}$, $B=\{6,8,9,10\}$, $C=\{7,8,9\}$
   a) $(A\backslash B)\backslash C$ b) $A\backslash(B\backslash C)$
2. Let $A=\{a,b,c\}$ Find all subset of A
3. Let $A=\{a,\{b,c\}\}$ Find all subset of A
4. Let $A\cup B=\{1,2,3,\{4\}\}$ and $A\cup C=\{1,2,\{3,4\}\}$
   Then Find a) $A\cup(B\cap B)$
5. Suppose A is proper subset of B, and Let $n(B)=6$
   Then the maximum possible element of:
   a) $n(A)$ b) $n(A\cap B)$ c) $n(A\cup B)$
6. Let $A=\{1,2,\{3,4\}\}$, $B=\{2,3\}$, $C=\{\{3,4\},5\}$
   Find a) $A\cap B$ b) $A\cap C$ c) $B\cap C$ d) $A\cap(B\cap C)$
7. If $A\subseteq B$, then. Find $A\,\Delta\,B=$ _____
8. If $A\cap B=A\cup B$, then. Find a) $A\,\Delta\,B$ b) $A\backslash B$
9. Given $A=\{1,\{2\},\{1,2\}\}$ and $B=\{1,\{1,2,3\}\}$ then find
   a) $A\cup B$ b) $A\cap B$ c. $A\backslash B$ d) $B\backslash A$
10. If $A=[-1,1]$ and $B=-2,0]$, then $A\backslash B$
11. If $A\subseteq B$, then. Find a) $B'\backslash A'$ b) $A'\,\Delta\,B'$
12. Let $A\backslash C=\{1,2,3\}$ and $B\backslash C=\{1,4,6\}$ then find $(A\cup B)\backslash C$
13. the simplified form of: a) $(B\backslash C)\cup(B\cap C)$ b) $(B\backslash C)\cup C$
14. $n(U)=12$, $n(A)=5$, and $n(B)=8$
    find a) $n(A')$ b) $n(B')$
15. Let $n(A\backslash B)=6$, $n(B\backslash A)=7$, and $n(A)=8$, then find
    a) $n(B)$ b) $n(A\cap B)$ c) $n(A\cup B)$
16. Answer each of the following. If your answer is No, tell why.
    a. If $A=B$, can we always conclude that $A\subseteq B$?
    b. If $A\subseteq B$, can we always conclude that $A\subset B$?
    c. If $A\subset B$, can we always conclude that $A\subseteq B$?

d. If $A \subseteq B$, can we always conclude that $A = B$?
e. $A \cup (B \cap C) = (A \cup B) \cap C$
f. $A \cap (B \cup C) = (A \cap B) \cup C$.

17. Determine which of the following sets are finite.
a. The set of lines parallel to the x-axis.
b. The set of letters in the English alphabet.
c. The set of animals living on the earth.
d. The set of circles through the origin

18. The Venn diagram in figure shows sets A, B and C. Simplify and shade the region the following sets.
a. $A \setminus (B \cup C)$ b. $(B \cup C) \setminus A$ c. $C \setminus (A \cup B)$



Answer and Explanation Review Exercise 3

1. a) {2, 3, 4, 5} b) {2, 3, 4, 5}

2. subset of A are ∅, {a}, {b}, {c}, {a,b}, {a,c} and {a,b,c}

3. subset of A are ∅, {a}, {a{b,c}}

4. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C) = \{1, 2\}$

5. a) $n(A) = 5$ b) $n(A \cap B) = 5$ c) $n(A \cup B) = 6$

6. a) $A \cap B = \{2\}$ b) $A \cap C = \{\{3,4\}\}$ c) $B \cap C = \varnothing$ d) ∅

7. $A \Delta B = B \setminus A$

8. a) $A \Delta B = \varnothing$ b) $A \setminus B = \varnothing$

9. a) {1, {2}, {1,2}, {1,2,3}} b) $A \cap B = \{1\}$
c) $A \setminus B = \{\{2\}, \{1,2\}\}$ d) $B \setminus A = \{\{1,2,3\}\}$

10. $A \setminus B = [0, 1]$

11. a) $B' \setminus A' = \varnothing$ b) $A' \Delta B' = (A' \setminus B) \cup (B' \setminus A')$
$= A' \setminus B' \cup \varnothing$
$= A' \setminus B'$

12. $(A \cup B) \backslash C = (A\ B)\ C' = C' \cap (A \cup B)$

$= (C' \cap A) \cup (C' \cap B)$

$= (A \backslash C) \cup (B \backslash C)$

$= \{1, 2, 3, 4, 6\}$

13. a) $(A \backslash B) \cup (B \cap C) = B - (B \cap C) + B \cap C = B$

b) $(B \backslash C) \cup C = B - (B \cap C) + C = B + C - (B \cap C) = B \cup C$

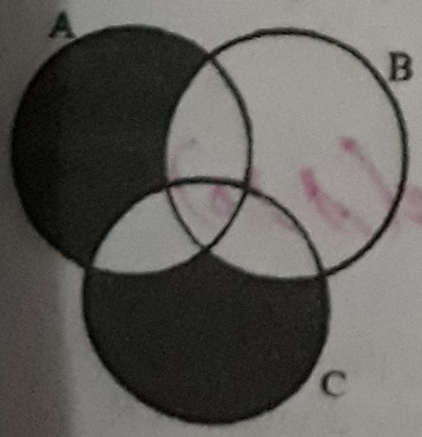
14. a) $n(A') = n(U) - n(A) = 12 - 5 = 7$, b) $n(B) = 4$

15. a) 9 b) 2 c) 15

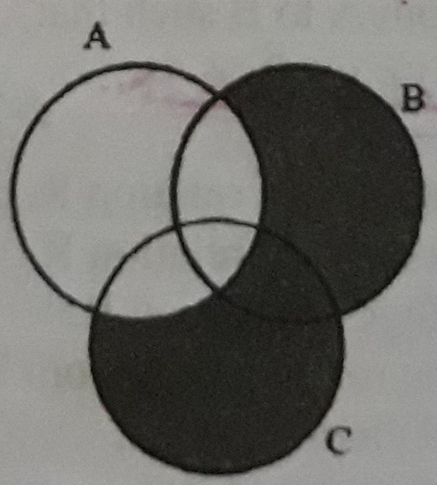
16. a. Yes
b. No, let $A = \{1, 2\}$, $B = \{1, 2\}$, the $A \subseteq B$, but $A \neq B$
c. Yes
d. No, Let $A = \{1, 2\}$, $B = \{1, 2, 3\}$ then $A \subseteq B$, but $A \neq B$
e. No, because $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
f. No, because $A \cap (B \cup C) = (A \cap B) \cup (A \cup C)$

17. a. infinite b. finite c. finite d. infinite

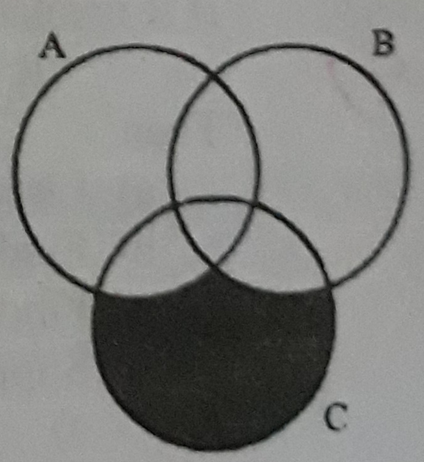
18.



$A \backslash (B \cup C)$ or $A \cap (B \cup C)'$



$(B \cup C) \backslash A$ or $(B \cup C) \cap A'$ or $(B \backslash A) \cup (C \backslash A)$



$C \backslash (A \cup B)$ or $C \cap (A \cup B)'$